Minigroupwork 4, solutions

a)

$$\rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 \quad \Rightarrow$$

$$r = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

Assuming the solar wind consists of protons

$$\rho_{SW} = n_{e,SW} m_p = 1.7 \cdot 10^{-22} \ kg \, m^{-3}$$

Thus

$r = 2.7\cdot10^9$ m ≈ 38 R_J

b)

$$\rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 + 2 n_e k_B T \quad \Rightarrow$$

$$\rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 + 2 n_{e0} \left(\frac{R_J}{r} \right)^3 k_B T$$

Substitute $x = 1/r^3$. This gives you an equation on the form

$ax^2 + bx + c = 0$

with

$$a = \left[\frac{\mu_0 a}{4\pi} \right]^2 / 2\mu_0 = \frac{\mu_0 a^2}{32\pi^2} = 1.0\times10^{46}$$

$$b = 2 n_{e0} R_J^{\ 3} k_B T = 3.6\times10^{18}$$

$$c = -\rho_{SW} v_{SW}^2 = -2.7\times10^{11}$$

The solution is

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}} = -1.8 \cdot 10^{-28} + \sqrt{3.24 \cdot 10^{-56} + 2.635 \cdot 10^{-57}} =$$
$$= -1.8 \cdot 10^{-28} + 1.87 \cdot 10^{-28} = 7.18 \cdot 10^{-30}$$

Then

$$r = x^{-\frac{1}{3}} = \left(7.18 \cdot 10^{-30}\right)^{-\frac{1}{3}} = 5.2 \cdot 10^{9} \text{ m} \approx 73\ \mathrm{R_J}$$